Exploramatics
Journal of Innovative
Engineering and Technology

Volume: 04 No: 02 | September-2021

ISSN (Online) 2636 – 590
ISSN (Print) 2636 - 591X

PREDICTING LIMIT CYCLES IN SYSTEMS WITH FRICTION USING DESCRIBING FUNCTION ANALYSIS TECHNIQUE

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Abstract - Most mechanical systems subject to friction are often difficult to analyse due to the non-linear nature of friction. One of the main effects of friction is the exhibition of limit cycle oscillations. Limit cycles are usually described as self-sustained oscillations of fixed amplitude and period resulting from system non-linearity. There has been some techniques used for predicting the existence of limit cycles in non-linear systems most of which are based on implementation of simple PID based position control and complex analysis. In this paper a simplified analytic approach for limit cycles prediction of a system subject to friction using describing functions technique is presented. The friction non-linearity is represented by a novel dynamic friction model with hysteretic features. The result of the describing function analysis method when compared to that of the PID controller showed strong similarity while implementation of the PID entails much design and resources thereby making it more expensive compared to the describing function method. The result also demonstrated the ability of the novel friction model to predict important non-linear friction features such as hysteresis with non-local memory and limit cycle oscillations.

Keywords: Pre-sliding gross-slide regimes, Frictionmodel, Non-linearity, Hysteresis with non-local memory, Describingfunctions method.

I. Introduction

Contacting mechanical systems experiencing a relativemotion are usually influenced by friction (Marton & Lantos, 2007), (Piatkowski, 2014).This friction feature is a nonlinear phenomenon and could be said to be the tangential force between two surfaces in contactwhen there is a relative motion between them (Márton, Fodor, & Sepehri, 2011). Generally, nonlinear systems such as saturation, relay and friction do not obey the principles of superposition and homogeneity unlike the linear systems(Nnaji, 2012). Hence the tools of analysis and control used for linearsystems are largely not effective for the analysis of nonlinearsystems. Nonlinear systems are usually characterized by someof these features;

- Possible existence of multiple equilibria
- Static gain variations in the stable non-linear systems due to different operating points.
- Exhibition of chaotic behavior
- Exhibition of Limit Cycle Oscillations (LCO), with fixed period and amplitude.

The friction non-linearity is often characterized by different features depending on the friction regime. The friction regimes are two, namely the gross sliding and pre-sliding(Buechner, et al, 2012), (Hsieh & Pan, 2000). In the pre-slide regime the friction is a function of the displacement between the contacting bodies. At this point there exists no relative motion between the surfaces in contact. In the other hand, in the gross-sliding regime, friction is a function of the relative velocity between the surfaces in contact. Here, there is relative motion between the bodies. Some of the more pronounced features of friction are; hysteresis with non-local memory(Dhaouadi & Ghorbel, 2008), frictional lag(Hess & Soom, 1990), stribeck effect (Stribeck, 1902), stick-slip motion(Lin, Yau, & Tian, 2013), chaos and non-drift property(Berger, 2002). As a result of these features, friction has adverse effect in most mechanical contacting systems with relative motion. Such effects like the limit cycle oscillation usually lead to position and velocity tracking and regulation errors(Johnson & Lorenz, 1991),(Peng et al., 2005), (Rizos & Fassois, 2009). Limit cycles are described as self-sustained oscillations of constant amplitude and period which could be stable or unstable. They are usually independent of the size of the initial conditions and are less sensitive to variations in the parameters of the systems.

In the field of control, modelling friction has been challenging due to its non-linear nature and its behavior strongly tied to the regime of friction. Many models of friction have been proposed for friction simulation and control. These models range from the simple static models such as the Coulomb friction model to the complex dynamic models such as the LuGre (Canudas de Wit, Olsson, Astrom, & Lischinsky, 1995), GMS(Lampaert, Al-Bender, & Swevers, 2003) and the novel dynamic friction(Nnaji, 2017) models. The static models are simple in implement however they are not able to capture friction dynamics such as frictional lag, hysteresis etc.,while the dynamic models capture friction dynamics to different degrees depending on the structure of the dynamic model. The ability of the LuGre to predict limit cycle oscillations is demonstrated in (Olsson, 1996). There is however no research known to the authors demonstrating the ability of the novel friction model for the prediction of limit cycles. Limit cycles prediction has often been accomplished through the use of accurate friction model deployed in a PID position control scheme or method of analysis using the describing functions approach or the phase plain method. In this paper both the analytical and PID position control methods were used to demonstrate the ability of the novel friction model to predict limit cycles and the results of both methods compared. The importance of limit cycle oscillations prediction is so as to be able to design appropriate control schemes for the mitigation of its effects in systems subject to friction.

The outline of the paper is as follows; section II explains theconcept of describing functions analysis and the novel friction model. Section III, limit cycle predictionusing the describing function analysis technique is demonstrated.In section IV a PID control example using the novel friction model is illustrated.Analysis and discussion of the results are made in section V.Section VI concludes the paper.

II. Describing Functions Concept

Nonlinear systems such as friction usually exhibit limitcycle oscillations. In this section the concept of describingfunctions analysis method is formulated for the prediction of limit cycles in systems with friction. The friction phenomenon in a system often exposessuch system to the negative effect, especially in the pre-slideregime. Since a closed form analytical solution of nonlinear differential equation is not easily obtained, an approximateapproach is usually sought as an alternative. One such methodis the describing function analysis. In this technique, a linear approximation of the nonlinearity is obtained using theextended frequency response technique. This approximation captures closely the nonlinearity and could therefore be used to predict the behavior of the nonlinear system.

A certainnonlinear system is shown in figure1 in which it is possible to separate both the nonlinear and linear parts of the system. If the linear part couldbe replaced by a low pass filter and the nonlinearity as a quasi-linearelement as in figure 1b, then the describing functions approach could be used for the analysis of such a system.

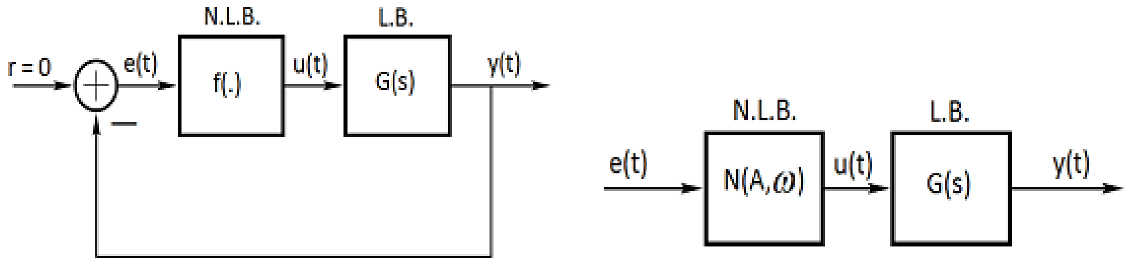

Fig. 1. Describing function analysis approach to non-linear systems; (a) Feedback loop of a system with hard non-linearity represented as f(.), (b) A Describing Function equivalent scheme.

The friction non-linearity can so be separated and thus the use of this method feasible. The outputof the nonlinear block represented as the functionf(.) and that of the describing function'sequivalent N(A, ω) can be approximately the same. For sustained oscillations and output fedback to the autonomoussystem, the output of the nonlinear block, assumed to beperiodic can be represented using a Fourier series. Usingthe output of the nonlinear block as input to the linear low-pass filter block ensures that only the fundamental frequencycomponent remains. Therefore, the Fourier series can be used to represent the output of the nonlinear block to a good degree ofaccuracy. So for the figure, if the input signal e(t) is given as

e(t) = Asinωt **(1)**

and the output of the non-linear block is

$u(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\big(a_n\ Cos(n\omega t) + bn\ Sin\ n\omega t$ **(2)**

with

$$a_n = \frac{2}{T}\int_0^T (u(t)\ Cos(n\omega t)\ d\omega t)$$

$$b_n = \frac{2}{T}\int_0^T (u(t)\ Sin(n\omega t)\ d\omega t)$$

$\omega$is the angular velocity, and n the nth term of the series.Assuming that the function is independent of frequency termω then the output is given as, (treating ω as a constant). From figure 1a

$u(t) = a_1 Cos(\omega t) + b_n\ Sin(\omega t)$ **(3)**

The output of the linear block LB will be

$y(t) = G(s)u(t)$ **(4)**

In terms of describing functions the output of the non-linear block NLBto any input e(t) is given as

$u(t) = N(A,\omega)\ e(t)$ **(5)**

where$N(A,\omega)$ is the describing function term. The objective however, is to obtain the output of the non-linear block to asinusoidal input of the form (equation 1) to be of the form (equation 2). Given that n = 1 i.e. the fundamental term of the series

then for n = 1,

$$u(t) = a_1 Cos(\omega t) + b_n\ Sin(\omega t)$$

which can also be expressed in terms of its magnitude and phase angle as

$$u(t) = M\ Sin(\omega t + \theta)$$

where the magnitude is $M(A,\omega) = \sqrt{{a_1}^2 + {b_1}^2}$

and the phase angle is $\theta(A,\omega) = \arctan(\frac{a_1}{b_1})$

Then using complex number notation

$u(t) = (b_1 + ja_1)e^{j(\omega t + \theta)}$ **(6)**

In terms of gain, the non-linear block NLB represented as $N(A,\omega)$ is the ratio of thefundamental output of the non-linear block $u(t)$and the inputsignal$e(t)$, we obtain

$N(A,\omega) = \frac{u(t)}{e(t)} = \frac{a_1 Cos(\omega t) + b_n\ Sin(\omega t)}{A\ Sin\ (\omega t)}$ **(7)**

Expressing this in complex notation yields

$N(A,\omega) = \frac{(b_1 + j a_1)e^{j(\omega t + \theta)}}{A\ e^{j\omega t}} = \frac{b_1 + j a_1}{A}$ **(8)**

From the above we have been able to deduce the describing functions gain equivalent$N(A,\omega)$,

Figure 1bof a non-linear block of the type f(.) of figure 1a, for a sinusoidal input signal e(t). For the particular case where the non-linearity is represented as the friction phenomenon modelled using the novel dynamic friction model described shortly, research has shown that the hysteretic friction behavior is not frequency dependent, so that the general describing function $N(A, \omega)$ can further be reduced to $N(A)$ for the particular case of friction hysteresis. The closed loop transfer function of the equivalent non-linear system of figure 1b is given in the Laplace transform as

$$T(s) = \frac{Y(s)}{E(s)} = \frac{G(s)N(A)}{1+G(s)N(A)} \quad \textbf{(9)}$$

wheres is the Laplacian variable.

In frequency domain terms the characteristic equation (i.e. the denominator of equation 9) is

$$1 + G(j\omega)N(A) = 0$$

Leading to

$$G(j\omega) = \frac{-1}{N(A)} \quad \textbf{(10)}$$

Equation 10 above is thus used for the prediction of the existenceand stability or otherwise of limit cycles in the given system.

The general approach for the prediction of limit cycles is as follows;

First obtain the Nyquist plot of the system transfer function $G(j\omega)$. Second is to plot the line graph of the reciprocal of the describing function term $N(A)$ on the Nyquist plot earlier obtained. If there is a point of intersection of the Nyquist and the reciprocal of the describing function gain then there exists a limit cycle oscillation. However, if the two plots do not meet at any point does not necessarily mean that there are no limit cycles. It simply implies that the describing functions method is not able to predict limit cycle existence for such system. This procedure will be further explained and implemented for the particular case of friction non-linearity in the subsequent sections following.

III. Describing Functions Method for Limit Cycle Prediction

One of the main applications of the describing functions analysis techniqueis in the prediction of the existence oflimit cycles in nonlinear systems and the nature of such limit cycles if they exist(Slotine & Li, 1991). Such non-linear systems could be a system with friction phenomena such as described in thispaper. Due to the sustained oscillating nature of these limitcycles, they can thus be represented by sinusoidal signals. In predicting the existence of limit cycles in friction systemsthe interest is usually in the pre-slide regime since it is the region with pronounced non-linearity. The friction force in thisregime is dependent only on the displacement and not therelative velocity of the moving surfaces.

The hysteresis friction force function with non-local memory characteristics presented in(Nnaji, 2017) is used for the limit cycle prediction performed in this paper. This hysteresis friction modelis multi-valued and as such its describing function will beexpressed as a complex function of the amplitude only sincethe friction hysteresis function is rate (ω) independent. Thepre-slide friction hysteresis function is given as

$$F_f(z) = Sin\left(\frac{z - z_r}{|z_t - z_r|}\frac{\pi}{2}\right)|F_t - F_r| + F_r \quad \textbf{(11)}$$

where $F_f(z)$ is the total friction force in the pre-slideregime at any given time, z the bristle displacementz_r the bristle displacement at the beginning of a branch, $z_t$is the target displacement (which is a function of the reversaldisplacement $z_r$and the breakaway displacementZ_b),F_r is the friction force at thebeginning of a branch (takes into account the stressed stateof the bristles), F_t target friction force (a function of thereversal point force F_r and the stiction force). This hysteretic friction model has been demonstrated to exhibit known features of friction both in the pre-slide and gross-slide regimes such as hysteresis with non-local memory, non-drift property etc.

The equation11 above is a function of the bristle deflection witha monotonically increasing characteristics. Other non-linearelements such as the hyperbolic tangent and some cumulative distribution functions with similar features as the sine functionused here could also be used. However the sine function wasused due to the ease of integrating relevant parameters of interest and attaining saturation in finite time as would bereal friction. The first term of equation 11 capturing thefrictional force for any branch and the second term the frictionforce value prior to external force application. Thus the secondterm is a description of the current state of the bristle before the external force influence. For simplicity

$$|F_t - F_r| = 2F_s$$

and

$$|z_t - z_r| = 2Z_b$$

with F_s and Z_b being respectively the stiction force and thebreakaway displacement and other terms being as previously defined. Using the simplification above equation 11 becomes

$$F_f(z) = 2F_s\, Sin\, \left(\frac{z - z_r}{2Z_b}\frac{\pi}{2}\right) + F_r \qquad \textbf{(12)}$$

This friction model of equation 12 above is therefore used to replace the non-linear function f(.) of figure 1a.

But the input signal $e(t)$ is a displacement signal such that

$$z(t) = e(t)$$

Therefore

$$F_f(z) = 2F_s\, Sin\, \left(\frac{e - z_r}{2Z_b}\frac{\pi}{2}\right) + F_r \qquad \textbf{(13)}$$

Given a sinusoidal input signal

$$e(t) = A\, Sin\, (\omega t)$$

to the non-linear function of equation 13, the friction is

$$F_f(z) = 2F_s\, Sin\, \left(\frac{A\, Sin\, (\omega t) - z_r}{2Z_b}\frac{\pi}{2}\right) + F_r \qquad \textbf{(14)}$$

where $z_r = \pm A$, and F_r the values of $F_f(z)$ at $\pm A$(this is the reversal point at which the displacement changes direction), A being the amplitude of the input signal.

Thus

$$F_r = F_s Sin\, \left(\frac{\pm A}{Z_b}\frac{\pi}{2}\right) \qquad \textbf{(15)}$$

and equation 14 becomes

$$F_f(z) = 2F_s\, Sin\, \left(\frac{A\, Sin\, (\omega t) - (\pm A)}{2Z_b}\frac{\pi}{2}\right) + F_s Sin\, \left(\frac{\pm A}{Z_b}\frac{\pi}{2}\right) \qquad \textbf{(16)}$$

From equation 16, it is clear that the nature of the output friction signal $F_f(z)$ is dependent on the nature of the amplitude of the input signal $'A'$ over a complete cycle in relation to the breakaway displacement Z_b. The 2 distinct scenarios emerge depending on if the input signal's amplitude is greater than the breakaway displacement parameter of the friction model.

Scenario 1) when the amplitude of the input signal $e(t)$ is greater than the breakaway displacement Z_b. That is $|A| > |Z_b|$

2) When the input signal $e(t)$ is less than or equal to the breakaway displacement Z_b.

Generally, over a complete period of displacement the friction force as given by the novel model due to the input displacement signal $e(t)$ is obtained from equation 17.

$$F_f(e) = \begin{cases} F_s & if\ e > Z_b \\ F_f(e) & if - Z_b \le e \le Z_b \\ -F_s & if\ e < -Z_b \end{cases} \qquad \textbf{(17)}$$

Instance 1: When $|A| > |Z_b|$

Recall the input signal equation 1 to be

$$e(t) = A\, Sin\, (\omega t)$$

Defining an angle $\emptyset_T$ such that

$$A\, Sin\, (\emptyset_T) = Z_b$$

So that

$$\emptyset_T = Sin^{-1}\frac{Z_b}{A}$$

By symmetry, the two halves of the period are same and by describing function analysis method, the output friction signal is obtained as

$$u(t) = F_f(e) = a_1 Cos(\omega t) + b_n\, Sin(\omega t)$$

The input being a displacement signal and the output a friction signal. The input and output relationship for this scenario is shown in figure 2.

This thus simplifies to

$$a_1 = \frac{2}{T}\int_0^T u(\tau)\, Cos\, (\omega\tau)\, d\tau \qquad \textbf{(18)}$$

with $u(\tau)$ replaced by $F_f(e)$ as represented in equation 17for a complete period.

Taking advantage of the symmetry overthe two halves of the period and simplifying gives

$$a_1 = \frac{4F_s}{A\pi}\left(\sqrt{1 - \frac{{Z_b}^2}{A^2}} - \frac{\sqrt{32}Z_b Sin\left(\frac{A\pi\sqrt{1 - \frac{{Z_b}^2}{A^2}}}{4Z_b}\right)}{A\pi}\right) \qquad \textbf{(19)}$$

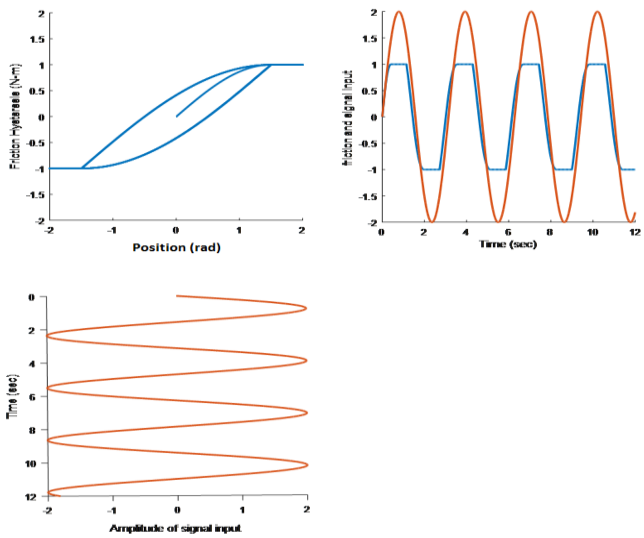

Fig. 2. Hysteresis friction for the condition that the signal input amplitude A is greater than the breakaway displacement Z_b; top-left; friction hysteresis, bottom-left; signal input and top-right; friction hysteresis and the signal input against time

In the same token

$$b_1 = \frac{2}{T}\int_0^T u(\tau)\ Sin\ (\omega\tau)\ d\tau \qquad \textbf{(20)}$$

Thus,

$$b_1 = \frac{4F_sZ_b}{A\pi} \qquad \textbf{(21)}$$

The describing function for the instance analyzed above is therefore given as

$$N(A) = \frac{b_1 + ja_1}{A} \qquad \textbf{(22)}$$

Substituting the right values obtained earlier yields the describing function to be

$$N(A) = \frac{4F_sZ_b}{A^2\pi} + j\frac{4F_s}{A^2\pi}\left(\sqrt{1-\frac{{Z_b}^2}{A^2}} - \frac{\sqrt{32}Z_b Sin\left(\frac{A\pi\sqrt{1-\frac{{Z_b}^2}{A^2}}}{4Z_b}\right)}{A\pi}\right) \textbf{(23)}$$

As expected the describing function equation 23 is a complex variablewhich is independent of the frequency of oscillation, and inagreement with the experimentally established fact that hysteretic functions with memoryusually have a complex describing function.Having derived the describing function equivalent for the friction non-linearity represented in the block of figure 1a, the next step is to use same to predict the existence or otherwise of limit cycles. To achieve this, one seeks to determine if there is an intersection of the plot of the line graph of the reciprocal of the describing function and the Nyquist plot of the transfer function $G(j\omega)$. The reciprocal of the describing function of equation 23 is then obtained as

$$\frac{-1}{N(A)} = \frac{\alpha + j\beta X}{\alpha^2 + (\beta X)^2} \qquad \textbf{(24)}$$

where

$$\alpha = \frac{4F_sZ_b}{A^2\pi}$$

$$\beta = \frac{4F_s}{A^2\pi}$$

and

$$X = \left(\sqrt{1-\frac{{Z_b}^2}{A^2}} - \frac{\sqrt{32}Z_b Sin\left(\frac{A\pi\sqrt{1-\frac{{Z_b}^2}{A^2}}}{4Z_b}\right)}{A\pi}\right)$$

The intersection of $\frac{-1}{N(A)}$as 'A' varies fromzero towards infinity and the Nyquist curve of $G(j\omega)$ asthe frequency changes from zero, if there is, establishes that the limit cycle oscillations existence,as well as the amplitude and frequency of the limit cycle in approximateterms.

Instance 2: When $|A| \leq |Z_b|$

For the situation where the amplitude 'A' of the input signal$e(t) = A\,Sin\,(\omega t)$is less than the breakaway displacementZ_b, this implies that the entire system motion dynamics iscontained in the pre-slide regime of friction and saturationnever occurs. This shows that for the entire cycle the signal isbounded between the positive and negative breakaway ($\pm Z_b$). This is captured in the friction equation 17 and the analysis is similar to that of scenario 1 except it never saturates (i.e. stiction F_s is never attained). As a result scenario 2 need not further study. The input and output relationship for this instance is shown in figure 3.

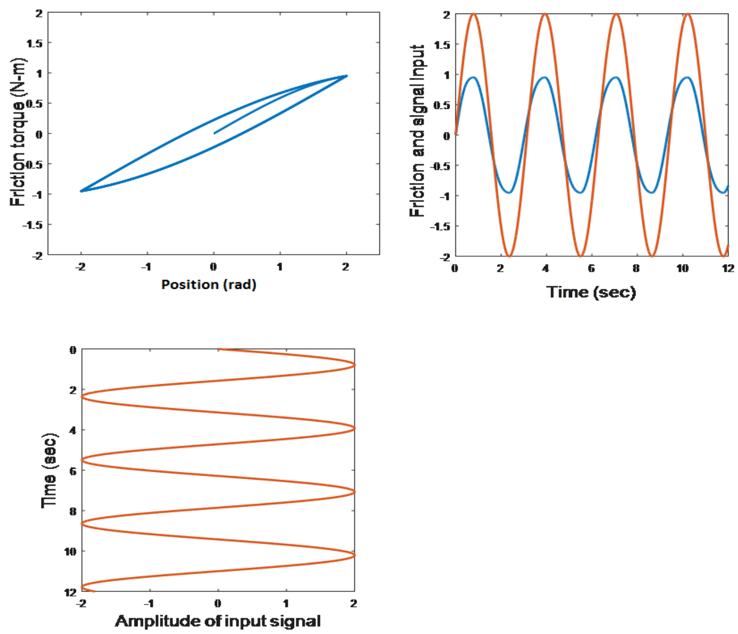

Fig. 3. Hysteresis friction for the condition that the signal input amplitude A is less than the breakaway displacement Z_b; top-left; friction hysteresis, bottom-left; signal input and top-right; friction hysteresis and the signal input against time.

Using describing functions analysis method and parameter values as presented in the next section for the prediction of limit cycle oscillations in mechanical systems with friction is shown in figure 4. That is intersection of the plot of the line graph of the reciprocal of the describing function (equation 24) and the Nyquist plot of the transfer function $G(j\omega)$.

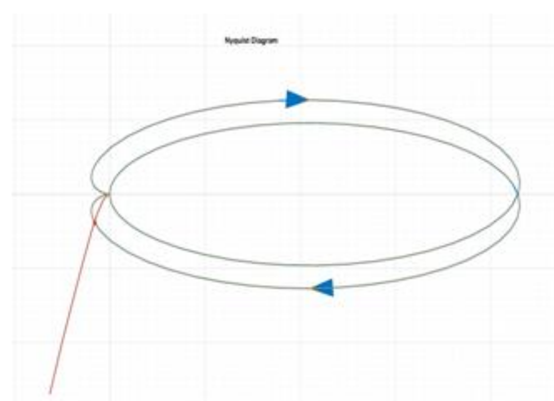

Fig. 4. Limit cycle oscillation prediction using the describing functionapproach showing the intersection of the Nyquist plot $G(j\omega)$ and the $\frac{-1}{N(A)}$ plot

III. Limit Cycle Prediction Using PID Position Controller

In the previous section we demonstrated the possibility of using describingfunctions analysis method for limit cycle prediction of system friction using the new hysteresis frictionmodel. In this section we consider an application example of the prediction of limitcycle oscillations by a system subject to friction on application of aPID controller.

Consider a certain system with the followingmotion equation;

$$m\ddot{x} + F_f(x) = u \qquad \textbf{(25)}$$

where m is the mass, x the displacement, $F_f(x)$ the systemfriction and u the control law. Applying a PID position controllerwith a control law u given as

$$u = K_v\dot{x} + K_p x + K_i \int_0^t (x(\tau) - x_{ref}(\tau))\, d\tau \qquad \textbf{(26)}$$

Where K_v is the derivative gain, $K_p$the proportional gain and K_i integral gain of the controller while x_{ref} is the referenceinput to the system.

The closed loop control performance of the system was observed with a reference input x_{ref} of 1m and a unit mass. For the case of a frictionless system, the control performance was good. However, the presence of friction quickly deteriorated the system performance and limit cycle oscillations are observed as shown in figure 5.

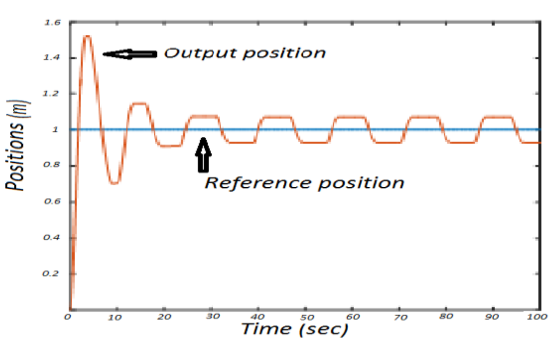

Fig. 5. Limit cycle oscillation prediction using a simulation approach

For K_p = 3, K_v = 6, K_i = 4, and m = 1, usingthe friction model [16] as the friction force $F_f(x)$present inthe system, the result of position control using the linearcontroller u above yields a position signal that oscillatesaround the reference position. Parameter values of the model used for the simulation are $Z_b = 0.001$, $F_s = 1$, $F_c = 0.6$, $v_s = 0.01$,$\sigma = 100$, $\tau = 0.002$.

The predicted amplitude using the describing functions method is A = 0.0787, and the limit cycle frequency of oscillation $\omega = 2.35\ rad/sec$, see figure 4while the amplitude A from the simulation is A = 0.0930 andthe corresponding frequency$\omega = 0.3927\ rad/sec$, see figure5. By this the capability of the proposed new friction modelto predict limit cycle oscillations is demonstrated. Fromfigure 4, it is observed that the predicted limit cycle is stablegiven that as A the amplitude increases, the value of $\frac{-1}{N(A)}$increases further away from the portion encircled by the$G(j\omega)$curve in the figure. The variations in the values of the predicted and actual values of the limit cycle oscillations isattributable to the fact that the describing functions method isan approximation technique and shows the effect of limiting the output only to the first (fundamental) harmonics.

IV. Presentation And Discussion of Results

From the graphs of figures 2 and 3, it is clearly demonstrated that the friction model presented is able to model true friction in the pre-slide regime. This hysteresis in the pre-slide is with non-local memory in that it does not forget its past history. The difference between the two results lies in the fact that in figure 2 the applied pre-slide displacement applied to the system with friction was larger than the breakaway displacement value and thus more than the stiction force. Beyond the breakaway displacement the input signal drives the system into gross-slide regime. At reversal points the friction force is seen to reverse thus tracing a hysteresis loop as shown in figure 2.This implies that beyond the breakaway displacement the friction force is not hysteretic in relation to the displacement while it is hysteretic in the region where the displacement is lower than the breakaway value.

In figure 3, the displacement value is always lower than the breakaway value and thus stiction is never reached. However at reversals the friction hysteresis changes direction and remembers the path it had traced previously. This is an important friction of friction and the LuGre model does not capture this true friction behaviour in the pre-slide regime. This model is therefore demonstrated to capture this feature of friction.

The describing functions analysis method result for prediction of limit cycle oscillations in mechanical systems with friction is shown in figure 4. In the figure the plot of the reciprocal of the describing function for the friction system and that of the Nyquist plot of the system function are seen to intersect each other thereby indicating the presence of limit cycle oscillations in the system. From the figure 4, the amplitude of the oscillations is obtained as 0.0787 and the oscillation frequency $\omega = 0.35$ rad/sec. Figure 4 further shows that the limit cycle oscillations so predicted is stable. This is so since as the amplitude is increased from zero, the value $\frac{-1}{N(A)}$ increases further away from the portion encircled by the Nyquist plot.

On the other hand the implemented PID based position control of systems with friction shown in figure 5 indicated the presence of limit cycles also. The amplitude of the limit cycles being 0.0930 and the corresponding frequency of 0.3927 rad/sec.

The output results of both approaches showed the presence of limit cycle oscillations in such systems with friction non-linearity, with constant amplitude and period. Both results of figures 4 and 5 indicate that the oscillation never comes to rest, so long as there is no external influence. The observed variations in the values of the amplitude and frequency of oscillations predicted by the describing functions method and the PID position control simulation in most attributed to the fact that the describing functions method is an approximate approach and shows the effect of limiting the output only to the fundamental harmonics. Due to the possibility of the system exhibiting limit cycles, the PID control method is rarely used alone for position control of system subject to friction non-linearity.

Despite its ability to predict limit cycles oscillations in friction systems represented by the novel friction model, the describing functions method has some limitations such as;

- Its operation is based on the assumption of a single non-linear element in the system. This implies that even if there is more than one non-linear element in the system, the most pronounced non-linearity is considered neglecting the rest. Where possible in systems with more than one non-linearity, lumping them into a single non-linear element is adopted.
- The system non-linearity is assumed odd. This ensures symmetric characteristics between the input-output relationships of the non-linear block about the origin.
- The system is unforced and time invariant, meaning there are no external inputs to the system and system parameters do not vary with time during operations.
- The linear part of the system provides sufficient low-pass filtering features such that only the fundamental Fourier series based output for the given sinusoidal input to the non-linear block is taken into account. This is also termed as the filtering hypothesis, since the filter-like linear blockwill ensure that higher order components are eliminated.

It is on account of these shortcomings that this method may sometimes yield inaccurate results.

V. Conclusion

The concept of describing functions analysis method for the prediction of limit cycles in systems with friction is presented. The approach uses the fundamental portion of the Fourier series to approximate the output of the non-linear system. A brief description of the novel dynamic friction model used to model friction hysteresis was also presented. The friction model was shown to be able to predict pre-slide friction hysteresis with non-local memory unlike the more popular LuGre model. Limit cycle oscillations prediction using the describing functions method was demonstrated and the results obtained compared with that of a PID based position control of a friction system. Both results showed the predicted limit cycles to be stable. The results showed that the variations in the values of the amplitude and frequency for both methods are quite close. The difference in the amplitude and frequency between the two approached is due to the describing functions method being an approximate method.

References

Berger, E. (2002). Friction modeling for dynamic system simulation. *Applied Mechanics Reviews*, *55*(6), 535. https://doi.org/10.1115/1.1501080

Buechner, S., Zschaeck, S., Amthor, A., Ament, C., & Eichhorn, M. (2012). Dynamic friction modeling and identification for high precision mechatronic systems. In *IECON 2012 - 38th Annual Conference on IEEE Industrial Electronics Society* (pp. 2263–2268). Ieee. https://doi.org/10.1109/IECON.2012.6388884

Canudas de Wit, C., Olsson, H., Astrom, K. J., & Lischinsky, P. (1995). A new model for control of systems with friction. *IEEE Transactions on Automatic Control*, *40*(3), 419–425. https://doi.org/10.1109/9.376053

Dhaouadi, R., & Ghorbel, F. (2008). MODELLING AND ANALYSIS OF NONLINEAR STIFFNESS, HYSTERESIS AND FRICTION IN HARMONIC DRIVES GEARS. *International Journal of Modelling and Simulation*, *28*(3), 329–336. https://doi.org/02329290320001151 37

Hess, D. P., & Soom, A. (1990). Friction at a Lubricated Line Contact Operating at Oscillating Sliding Velocities. *Journal of Tribology*, *112*(1), 147. https://doi.org/10.1115/1.2920220

Hsieh, C., & Pan, Y.-C. (2000). Dynamic behavior and modelling of the pre-sliding static friction. *Wear*, *242*(1–2), 1–17. https://doi.org/10.1016/S0043-1648(00)00399-9

Johnson, C. T., & Lorenz, R. D. (1991). Experimental identification of friction and its compensation in precise, position controlled mechanisms. In *Conference Record of the 1991 IEEE Industry Applications Society Annual Meeting* (pp. 1400–1406). IEEE. https://doi.org/10.1109/IAS.1991.178043

Lampaert, V., Al-Bender, F., & Swevers, J. (2003). A generalized Maxwell-slip friction model appropriate for control purposes. In *IEEE International Workshop on Workload Characterization (IEEE Cat. No.03EX775)* (Vol. 4, pp. 1170–1177). IEEE. https://doi.org/10.1109/PHYCON.2003.1237071

Lin, C., Yau, H., & Tian, Y. (2013). Identification and Compensation of Nonlinear Friction Characteristics and Precision Control for a Linear Motor Stage. In *IEEE Transactions on Mechatronics* (Vol. 18, pp. 1385–1396).

Márton, L., Fodor, S., & Sepehri, N. (2011). A practical method for friction identification in hydraulic actuators. *Mechatronics*, *21*(1), 350–356. https://doi.org/10.1016/j.mechatronics.2010.08.010

Marton, L., & Lantos, B. (2007). Modeling, Identification, and Compensation of Stick-Slip Friction. *IEEE Transactions on Industrial Electronics*, *54*(1), 511–521. https://doi.org/10.1109/TIE.2006.888804

Nnaji, A. C. (2012). *Introduction to control systems*. Enugu, Nigeria: De-Sanctity Publications.

Nnaji, Anthony Chidolue. (2017). *Modelling and control of velocity and position drives subject to friction*. University of Reading, United Kingdom.

Olsson, H. (1996). *Control Systems with Friction. PhD thesis*. Lund Institute of Technology (LTH).

Peng, K., Chen, B. M., Member, S., Cheng, G., Member, S., & Lee, T. H. (2005). Modeling and Compensation of Nonlinearities and Friction in a Micro Hard Disk Drive Servo System With Nonlinear Feedback Control. *IEEE TRANSACTIONS ON CONTROL SYSTEMS TECHNOLOGY*, *13*(5), 708–721.

Piatkowski, T. (2014). Dahl and LuGre dynamic friction models — The analysis of selected properties. *Mechanism and Machine Theory*, *73*, 91–100. https://doi.org/10.1016/j.mechmachtheory.2013.10.009

Rizos, D. D., & Fassois, S. D. (2009). Friction Identification Based Upon the LuGre and Maxwell Slip Models. *IEEE Transactions on Control Systems Technology*, *17*(1), 153–160. https://doi.org/10.1109/TCST.2008.921809

Slotine, J.-J. E., & Li, W. (1991). *Applied Nonlinear Control*. Englewood Cliffs, New Jersy 07632: Prentice-Hal Internationall, Inc.

Stribeck, R. (1902). Die wesentlichen eigenshaften der gleit-und rollenlager- The Key Qualities of sliding and roller bearings. *VDI-Zeitschrift Seutscher Ingenieure*, *46*(38, 39), 1341–1348, 1432–1438.